DOI: 10.53660/CLM-4298-24V03

Dewatering of sinter feed as a bulk draining process

Desaguamento de *sinter feed* como processo de drenagem de granel

Received: 20-09-2024 | Accepted: 21-10-2024 | Published: 24-10-2024

Altieres Marçal Frade
ORCID: https://orcid.org/0000-0003-3965-7256
Vale S. A., Brazil
E-mail: altieres.frade@vale.com
José Aurélio Medeiros da Luz
ORCID: https://orcid.org/0000-0002-7952-2439
Federal University of Ouro Preto — UFOP, Brazil
E-mail: jaurelio@ufop.edu.br

ABSTRACT

Screen dewatering is widely used in ore dressing operations. A deep understanding of its intrinsic mechanisms can enable the improvement of economic and even mitigation of environmental impacts. The dewatering of iron ore fines has been approached by different ways in recent years. Several authors consider that moisture decrease is essentially linked to residence time, while others attempt to treat this process via parametric modeling. This work explores the comparison between the van Genuchten's porous media dewatering model and the King's dewatering model, in order to make these two independent models compatible. This approach has the potential to overcome the difficulties of existing approaches by providing a useful and robust modelling alternative.

Keywords: Dewatering; van Genuchten; Iron ore; Stockpile; Screening.

RESUMO

Desaguamento em peneira é amplamente utilizado na indústria mineral. A plena compreensão de seus mecanismos intrínsecos pode possibilitar diminuição de impactos econômicos e mesmo ambientais. O desaguamento de granéis de minério de ferro tem tido diversas abordagens nos últimos anos. Algumas delas consideram o decréscimo de umidade como fenômeno essencialmente ligado ao tempo de residência nos equipamentos que realizam este processo. Outras tentam tratar esse processo com uma modelagem paramétrica. Para tentar superar as dificuldades na determinação dos parâmetros destes modelos, este trabalho explora a comparação entre o modelo de desaguamento em meio poroso de van Genuchten, e o modelo de desaguamento de King. Esta abordagem tem o potencial de superar as dificuldades das abordagens existentes, fornecendo uma alternativa de modelo útil e robusta.

Palavras-chave: Desaguamento; van Genuchten; Minério de ferro; Pilha de granel; Peneiramento.

INTRODUCTION

Iron ore dewatering is a key step in mineral beneficiation, as it aims at decreasing saturated water and the moisture content of the final product to make possible its handling, storage and transportation, mainly by shipping. Moreover, dewatering contributes to water and energy savings, as well as to the minimization of environmental impacts associated with tailings disposal.

One of the most widely used techniques for iron ore dewatering is dewatering screens. These screens are machines that have a deck surface that allows the passage of free water and retains the solid particles. The vibration of the screen generates an inertial force that acts on the particles and promotes the drainage of the interstitial water between them. This is usually inferred by a dimensionless ratio called G-factor or gravitational force equivalent (or simply G-force), which is defined as the ratio between the maximum acceleration of the screen and the acceleration of gravity ($\boldsymbol{G = g \cos\theta_{scr}/A\omega^2}$), being $\boldsymbol{\theta}$ the angle of the dewatering screen, $\boldsymbol{A}$ the amplitude and $\boldsymbol{\omega}$ the angular frequency of the vibration (Milhomen, 2013).

The moisture content of iron ore after screening depends on several factors, such as: the characteristics of the ore (particle size, shape, density, porosity, surface wettability, etc.), the operating conditions of the screen (frequency, amplitude, inclination, screening time, etc.) and the properties of water (viscosity, surface tension, etc.). Therefore, it is important to know the relationship between the moisture content of iron ore and the *G*-force applied by the screen, in order to improve the dewatering process and obtain the lower moisture as possible. Additionally, the efficiency of removing residual moisture from bulk materials like iron ore can be enhanced by adjustment of mineral surface wettability and/or lowering of water surface tension.

Patra et al. (2016) utilized a variety of reagents in their research to improve the iron ores fines dewatering efficiency. Adding surfactants, they have reached final moisture reduction from 13 – 12 % to 10 – 9 %. In an investigation of hydrophobicity effect on draining of interstitial water from bulk piles (iron sinter feed and coal). Luz (1992; 1993) tried petroleum sulfonate or carboxylates (derived from the saponification of vegetable oils) to accelerate the bulk dewatering. He found that the addition of these surfactants typically resulted in a reduction of two percentage points in final moisture, compared to piles without surfactant additives (blank tests).

Van Genuchten equation (Mello *et al.*, 2005; Han, *et al.*, 2009; Andrade & Stone, 2011; Rudiyanto *et al.*, 2015; Zhang *et al.*, 2022) is used to represent the relationship between matric potential and volumetric water content in soil. That equation has five parameters ($\boldsymbol{\theta_s}$, $\boldsymbol{\theta_r}$, $\boldsymbol{n}$, $\boldsymbol{m}$ and $\boldsymbol{\alpha}$) that can be adjusted to experimental data by means of nonlinear regression. The parameters $\boldsymbol{\theta_s}$ and $\boldsymbol{\theta_r}$ represent, respectively, volume moisture content at saturation and at residual condition, while parameters $\boldsymbol{n}$, $\boldsymbol{m}$ and $\boldsymbol{\alpha}$ are related to fit the curve shape.

Generally speaking, the volumetric water content ($\boldsymbol{\theta}$) is a volume ratio obtained by the expression:

$$\theta = \frac{V_w}{V_t} = \frac{V_w}{V_s + V_p} = \frac{V_w}{V_s + V_w + V_g} \tag{1}$$

Where: $\boldsymbol{V_w}$, $\boldsymbol{V_t}$, $\boldsymbol{V_p}$, and $\boldsymbol{V_g}$ are, respectively: volume of water, total volume, volume of pores, and volume of gas (air). This parameter is actually different of the so-called saturation fraction, $\boldsymbol{\Phi_{sat}}$, which is the water to pores volume ratio.

In fact, in the field of ore treatment, the most used parameter is mass-based moisture ($\boldsymbol{u}$), rather than volumetric water content. These two properties are related by the following expression:

$$\theta = \frac{u \times (1 - \epsilon) \times \rho_s}{(1 - u) \times \rho_l} \tag{2}$$

The analogous approach proposed in this work allows estimating moisture content for iron ore for a given G-force, as well as evaluating influence of ore characteristics and operating conditions of screen on dewatering performance.

THEORETICAL BACKGROUND

Dewatering principle

The principle of dewatering by screen follows principles very close to those of mineral slurry filtration. The principles of fluid flow in porous media are modeled by Darcy's law . This law determines that the quantity of fluid depends on variables such as the bed thickness, the permeability of the bed and the pressure gradient to which the bed is subjected.

Regarding dewatering screens, the concept of permeability helps to explain the flow rate of filtrate (the liquid that percolates both the filter medium elements and the particulate bed itself). If there is a higher permeability, the easier it is for liquid to flow through the porous medium. This parameter is a function of the fluid's viscosity, the screen area, the pressure range between surfaces and depends on the morphometric properties of the particles that make up the bed.

Quites (2018) studied the dewatering of iron ore from the so-called Iron Quadrangle (Brazil), using the gravity drainage process. The ore, from a mine located in Minas Gerais State, was represented by six samples of different types of rocks. The results of this study showed that mineral assemblage and particle size distributions are critical factors to residual moisture response.

Rivera *et al.* (2017) discussed the wettability of seven iron ore samples in granulation process (preparation for feeding sintering operations). The authors' conclusion was the origin of the iron ore has a major impact on modulation of the particle wettability.

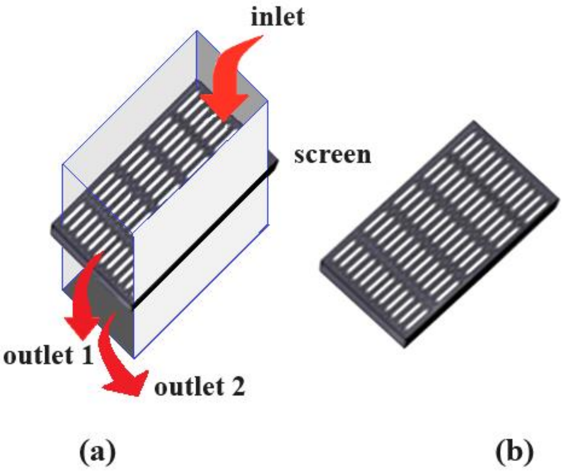
Screening can act as a mechanical dewatering system, in solid-liquid separation applied to mineral slurries. The process can be treated as a granular flow segregation. The solids content in the feed is typically around 65% by mass, in case of iron ore beneficiation.

Zhu *et al.* (2012) proposed a model that considerer two water movements during the dewatering process, one concerns the flow through the particulate bed, while the other concerns the flow through the screen itself. These authors have applied this conceptual model in a shale shaker (an intensive vibrating screen), similarly to Benis (2017), who divides the shale shaker into two contiguous domains, one for the cake formation stage, while the second involves cake drainage.

Raja *et al.* (2010) in their study mention there is two sections to be considered when the dewatering process is running parallel to the deck surface, in the first section of cake formation (where occurs the filtration), and there is the drainage section, in which occurs the liquid drainage.

As the water trapped in the interstitially porous media flows with the bulk over the deck the equipment and contributes to the final moisture content of the bulk material (outlet 1 in Figure 1). Meanwhile, outlet 2, from Figure 1, collects the percolated liquid, which may contain ore small particles, or ore fines.

Figure 1 – Models are considered for the implementation of a dewatering simulation on computer algorithms. In the figures below are showed models for the fluid percolation and interstitial flow on a sieving surface (a) and a screen model (b)



Source: Authors' elaboration (2024)

This downward liquid passes through the very small openings. An example of the size of these screens can be taken from iron ore (sinter feed) dewatering, which generally uses rectangular apertures 12 mm long by 0.3 mm wide.

In a continuous and steady-state system, a mass balance in terms of the moisture content at the inlet and outlet of the dewatering screen allows the water volumetric flow rate (**Q_{vu}**, in m³/s) in the screen's underflow to be calculated:

$$Q_{vu} = \frac{Q_s}{\rho_l} \times \frac{(u_{in} - u_{fin})}{(1 - u_{in}) \times (1 - u_{fin})} \tag{3}$$

Where: **Q_s** — solid mass flow rate [kg/s]; **u_{fin}** — final moisture of cake in kg of water per kg of wet cake [–]; **u_{in}** — initial moisture of the slurry [–]; **ρ_l** — density of liquid (water) [kg/m³].

As pointed out by Keller and Stahl (1994b), efficient dewatering process can be achieved by using different machine that impose different levels of acceleration on particles. Those authors mention centrifuges and vibrating screens. Centrifuges reach accelerations of 80 times that of standard terrestrial gravity, or even more, while dewatering screens can reach up to 10 times this reference value.

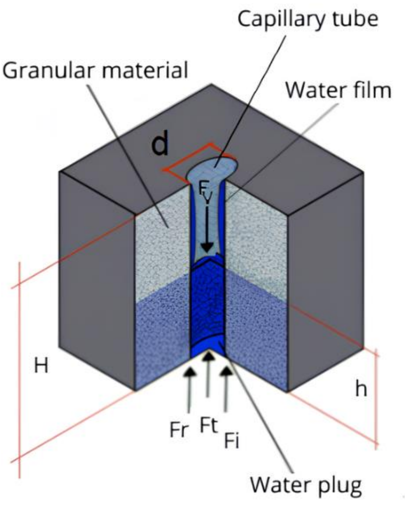
King (2001) proposed a set of equations to describe the water unsaturation condition (which he somewhat inappropriately calls “saturation”) of a granular system leaving a dewatering screen. This water saturation mass fraction has a dimension of kg of water per kg of slurry. As a matter of fact, this method has resulted in good predictability for coal dewatering, after parameter calibration from experiments. Actually, the model parameters depend on conditions like porosity, permeability and bed compressibility.

Li et al. (2021) developed a dewatering model based on a combination of computational fluid dynamics (CFD) and finite element methods to predict screening and dewatering efficiency. This model's output was compared with the results of a physical dewatering model under the same parameter conditions using a slurry containing quartzous sand with grain size range between 0.6 mm and 3.0 mm. The simulation results were somewhat different from those obtained in experiments using the physical model.

Dewatering model

The basic premise here is that the moisture evolution of the granular material during dewatering is a monotonically decreasing function of the residence time on the screen. Furthermore, the technique encompasses two intrinsic stages. The first one aims at determining how long the ore is exposed to dewatering. The other one aims to assess the influence of the system (equipment features and granular material properties) on the dewatering rate. As illustrated in **Erro! Fonte de referência não encontrada.**, Keller et al. (1994) introduced a model to represent the phenomenon, contrasting the frictional and vibratory forces involved in the dewatering system under a gravitational field (or under an inertial equivalent one).

Figure 2 – Dewatering model configuration after Keller et al. (1994), with frictional and vibrational forces



Source: Authors' elaboration (2024)

Erro! Fonte de referência não encontrada., adapted from Keller et al. (1997), represents the water flow into de bulk material. Water flows downward in the bulk material, creating a saturated layer with thickness $\boldsymbol{h}$. Initially, this layer was as thick as the particulate bed itself ($\boldsymbol{H}$). One way to modelling this process can be by a capillary tube. This is a theoretical tube that have an average diameter related with of the porous and that the water flow inside the granular material. Concerning the water film, Rudiyanto *et al.* (2015) conclude that water retention parameters would be related to the film conductivity parameters by empirical modeling.

The model from **Erro! Fonte de referência não encontrada.** shows the forces involved in the dewatering process. The force of inertia, friction and capillary force (resulting from the liquid's surface tension) are the fluid flow resistance. The vibrational force is against the resistance and can initiate the dewatering process. Keller *et al.* (1997) adopt the “kinetic parameter” as a dimensionless ratio between resistance force and vibrational force to measure how dewatering runs during the vibrational screening.

$$\lambda = \frac{2 * \eta * d}{(g * C * h^2)} \tag{4}$$

Where: $\boldsymbol{\lambda}$ — kinetic parameter [–]; $\boldsymbol{\eta}$ — dynamic viscosity of water [Pa.s]; $\boldsymbol{d}$ — capillary effective (or apparent) diameter [m]; $\boldsymbol{g}$ — gravity acceleration [m/s²]; $\boldsymbol{C}$ — equipment acceleration [m/s²]; $\boldsymbol{h}$ — water thickness layer [m].

A mechanical system (screen) should be taken into account to determine the residence time of the bulk on dewatering screen. This quantity can be calculated due the physical dimension (length) and constant deck velocity produced by forces. These forces are induced by the equipment drive.

Vector forces decomposition makes it possible to establish a gravitational parameter, throwing number or throwing index (frequently — and inappropriately — called “G-force”), which is the ratio between the resultant vector perpendicular to the dewatering surface and the weight force (modified from King, 2001; Heng-Shen et al., 2002; Stoicovici et al., 2009).

$$G_y = \frac{\omega^2 \times A_y}{g \times cos\theta} = \frac{\omega^2 \times \mathrm{A} \times \sin \beta_v}{g \times cos\theta_{sc}} \tag{5}$$

Where: G_y — throwing number (coefficient of throwing) in y direction [–]; g — gravity acceleration [m/s²]; ω — angular velocity [rad/s]; A_y — oscillation amplitude [m]; A_y — oscillation amplitude in normal direction (y) [m]; β_v — angle of inclination of the inertial force vector with respect to the screen surface [–]; Θ_{sc} — screen declination (dip angle of screen) [–].

The velocity through the screen can be given by the equation below (modified from King, 2001), which displays this function between the gravitational parameter and empirical coefficients:

$$v_s = a \times \omega \times A_x \times exp\left(-\frac{b}{G_y}\right) \tag{6}$$

Where: A_x — amplitude of vibration in the direction parallel of the screen surface [–]; a — specific material velocity parameter [–], b — specific parameter depending on the screen angle (Θ_{sc}). King (2001) states that the parameter b can be obtained by

$$b = 1.44 - 2.32\ \Theta \tag{7}$$

Ettmayr *et al.* (2000) point out that the detachment (or lift off) time is a function of frequency, amplitude, and screen angle. Anyway, the time that ore cake travels along the screen is the product of speed and length of the equipment.

$$t = \frac{L}{v} \tag{8}$$

Where: t — time [s]; L — screen length [m]; v — material travelling speed [m/s]. For its part, the dewatering rate (derivative of moisture, u, as a function of time) can be expressed by Equation 7 (according to King, 2001), where u_{eq} is the limit moisture at equilibrium (for a hypothetically infinite residence time).

$$\frac{du}{dt} = -\alpha \times \left(u - u_{eq}\right)^{\beta} \tag{9}$$

By Integrating (by separation of variables) the previous equation, from initial moisture (u_{in}, at $t = 0$) to the final moisture (u_{fin}, at the final t, and being, and $u_{in} > u_{fin} >> u_{eq}$), one gets:

$$\frac{1}{(1-\beta)} * \left[\left(u_{in} - u_{eq}\right)^{1-\beta} - \left(u_{fin} - u_{eq}\right)^{1-\beta}\right] = -\alpha * t \tag{10}$$

Therefore, the moisture expressed as a function of time (actually, the residence time) results:

$$u(t) = \sqrt[(1-\beta)]{\alpha * (\beta - 1)t + \left(u_{in} - u_{eq}\right)^{1-\beta}} + u_{eq} \tag{11}$$

u(t) – moisture in given time, in kg of water per kg of slurry (solids plus remanent water) [–]; u_{fin} — final moisture operationally achievable [–]; $u_{in} = u(t{=}0)$ — initial moisture of the bulk [–]; u_{eq} — minimum achievable moisture content (if the dewatering device had an infinite length, or the dewatering time were, so equally, infinite) [–]; α and β are constants (provided evaporation during dewatering is negligible) [–]; t — dewatering time [s].

Following King (2001), the parameters α and β can be find through the relation below,

$$q = \frac{1}{(\beta - 1)} \tag{12}$$

$$p = \left(\alpha(\beta - 1)\right)^{q} \tag{13}$$

This allow to find the residence time, bulk velocity and the dewatering screen length which are needed to reach the target moisture.

Wang *et al.* (2010) studied dewatering screens using computational fluid dynamics (CFD). One of the main results found was that the fluid can take different flow directions due to the speed of the equipment, sometimes flowing in the direction of the screen surface and sometimes in the opposite direction.

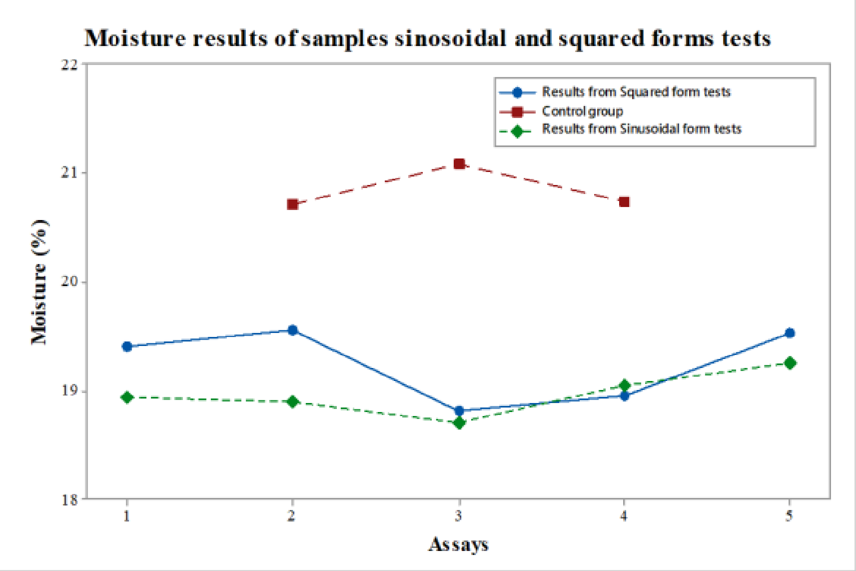
Zhu et al. (2012) developed a new model for dewatering clay suspensoids based on the kinematic equations of the liquid moving along and across the screen deck, as well as the equation of mass conservation.

Raja et al. (2010) carried out experiments to determine a dewatering model for clay suspensoids. These studies showed particle diameters and porosities have considerable effects on dewatering performance, but the inclination of the screen does not have a significant effect as long as the bed porosity is kept constant.

In turn, Ng *et al.* (1990) studied dewatering process of coal under various conditions of oscillating machine and ore parameters. Their conclusion was that for the same ore characteristic, dewatering performance is improved for large amplitudes.

Frade and Luz (2022), through various industrial dewatering tests of iron ore with sinter feed granulometry, evaluated the influence of the continuous variation of the equipment's frequency on the moisture result of this dewatering process. The authors found that the application of a continuously modulated frequency to the screen resulted in lower moisture levels compared to the application of a fixed oscillatory frequency. Figure 3 shows effect of dynamical frequency variation in industrial dewatering screen, applying two kinds of oscillation pattern (full circles in blue are for square wave or stepped tests; full rhombs ("diamonds") in green are for sinusoidal wave; and red full squares refer to industrial operation — the control group —, with fixed nominal frequencies). In that campaign aliquots of the product were collected every 5 minutes. Each three aliquots made up one sample. Each three aliquots made up one sample, resulting, at the end of the tests, five samples for each vibration pattern.

Figure 3 – : Moisture of dewatering iron ore sinter feed, changing the oscillating pattern



Source: Authors' elaboration (2024)

Tschapek et al. (1985) studied the reasons for the existence of undrainable water in quartz and glass particle granules using a dewatering column. The authors considered that the existence of undrainable water in these granules is due to the factors of particle size and the surface tension of the water.

Guerreiro *et al.* (2015) tested the impact of operational parameters on shale dewatering. The screen used had two 0.55 kW vibrators equipped with a frequency inverter and a piezoelectric accelerometer. Such an approach used a statistical design of experiments with three factors, being type of screen, volume concentration (***C_v***) and G-force on the screen. The results have shown that moisture reduces when stroke has low ***G*** and ***C_v***.

The van Genuchten equation

Studies of soil moisture behavior often rely on van Genuchten's soil characteristic curve (Mello, 2005). The relationship which is used to obtain this curve is using the water content on a volume basis (***θ***) instead of a mass basis (***u***):

$$\theta = \theta_r + \frac{(\theta_{sat} - \theta_r)}{(1 + (\alpha_1 \times |\psi|)^n)^m} \tag{14}$$

Where: ***θ*** —predicted volumetric moisture after the equilibrium under a suction head [m³/m³] = [–]; ***θ_r*** — residual volumetric water content [–]; ***θ_{sat}*** — saturated volumetric water content [–]; ***ψ*** — matric suction potential [m]; ***α_1*** — pressure head over the soil [1/m]; ***n*** — parameter linked to the distribution of the pores [–]; ***m*** — pore asymmetry [–].

The relation between ***n*** and ***m*** is given by (Mualen, 1976):

$$m = 1 - \frac{1}{n} \tag{15}$$

Related to the concept used by van Genuchten is the idea of the material potential presented by the particles that make up the soil. Matric potential is related in the equation for the total potential of water in a soil matrix. This total potential is an addition relationship between the main potentials presented in the soil and is a way of characterizing how water flow through the soil. These potentials are pressure, matric and gravitational.

$$\psi = \psi_{matric} + \psi_{pressure} + \psi_{gravity} \tag{16}$$

Mello et al. (2005) demonstrated that it is possible to predict variables in the van Genuchten equation parametrically using characteristics determined by soil samples, without laboratory assays.

It would be possible to consider that van Genuchten could easily be applied to the condition of dewatering in piles (dump dewatering), since dewatering takes place under the action of gravity (***G*** = ***a_{max}/g*** = 1.0, where: ***g*** = 9.81 m/s² and ***a_{max}*** is the normal maximum acceleration), while dewatering carried out on dewatering screens takes place under the action of forces greater than g (i.e. ***G*** > 1.0). However, the draining of water in a dewatering pile is continuous, whereas the draining in a dewatering screen is discontinuous. Furthermore, it should be noted that the effect of interstitial liquid drainage in granular media incorporates the algebraic sum of inertial forces (accelerative and/or gravitational) and forces arising from capillary pressures, including those affected by the existence of non-planar particle/liquid interfaces (with convexity or concavity, or even Plateau edges), forces quantifiable by the Young–Laplace equation.

Han *et al.* (2010) have estimated water retention curve from soil (Loess Plateau of Chine) with van Genuchten with a good precision. They assess the precision with statistic parameters like RMSE e NRMSE. Yue *et al.* 2022 observed when the temperature rises the water retention capacity reduces. Moezzibadia *et al.* (2019) applied numerical approach to understand how the van Genuchten parameters evolve during rain-fall periods.

Lozano *et al.* (2020) used centrifugal action to obtain de water retention curve through a centrifuge based on van Genuchten equation.

The King's method

The equation modified from King (2001) considers the rapid acceleration, and deceleration of the water cause the water drainage leads this fluid to the screen deck became possible the drip condition. Below the modified equation from King that considers time as a process variable.

$$u(t) = \sqrt[(1-\beta)]{\alpha * (\beta - 1)t + \left(u_{in} - u_{eq}\right)^{1-\beta}} + u_{eq} \tag{17}$$

That function is the way to forecast the moisture of the bulk in a given time in process. The parameters $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ could be reached by the relations of Equation 12 and 13, respectively, from the parameters ***p*** and ***q***, as previously shown. And in turn, these parameters could be related to the, respectively, the solids to be dewatered, and the condition the sieves on the screen (King, 2001).

That modified equation offers important link among time, water dynamics and equipment features to improve the dewatering process, but there is a lack of information on the important parameters linked to the material properties which can supply more accurate knowledge about bulk dewatering.

RESULTS AND DISCUSSION

As we have discussed before, we will adopt two references to develop the relationship between the King and van Genuchten methods. We have jointed these two models to construct a third modified equation with will better explain the process of dewatering of the bulk materials.

Both equations are empirical and are used for final moisture prediction in a densified granular media.

The time-dependent equation showed by King (2001) is related to many parameters as dewatering relation ***q, p*** and form factors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$. With these terms in this equation try to describe the dewatering in an industrial screen where force fields can reach till 30 times of the gravity (in case of centrifuges). It was used as a way to predict the six parameters from van Genuchten equation, namely: residual and saturated volume water content, pressure at a reference level, pore size distribution, pore asymmetry and matric potential.

In iron ore industry this approach can be applied for predict both moisture content in stockpiles, under effective $\boldsymbol{G} = 1.0$ (only gravity), and for industrial dewatering process under several values of earth acceleration ($\boldsymbol{G} >> 1.0$).

Unifying the two approaches

One way to develop a new calculation procedure with both methods is using the van Genuchten as predictor of the ultimate equilibrium moisture, when a matric potential (here linked to ***G***) is applied, provided no evaporation occurs. After that prediction, King equation can forecast the moisture in the industrial dewatering screen.

In case of only the natural gravitational field acting on de bulk, the van Genuchten term in the new equation can predict dewatering rate while the bulk stays as stockpile, or during the hauling, when the gravity is dominant and the water flow continuously (at least at first approach). On the other hand, under induced field forces the King method can forecast the final moisture.

To integrate both models within the context of the van Genuchten matric potential and consider the oscillatory effects on a dewatering screen, we evaluated the influence of the screen on the dewatering process. The central hypothesis was that the screen induces a constant alteration in the gravitational reference — predicted in soil science — which can oscillate up to twice the oscillation amplitude of the dewatering equipment.

Based on this oscillation, we use a factor to multiply the matric factor by a factor of $\boldsymbol{g}/\boldsymbol{G}$ (Equation 18), when under the influence of the equipment. It is important to emphasize that, due to the nature of the screen, which is limited to throwing the particles, its influence on the fluid displacement, according to van Genuchten, is limited to situations where ***G*** is equal to one. In order to avoid this effect on van Genuchten during industrial dewatering we introduce the term below.

$$\cos\left(\frac{g}{G_y}\right) = \cos\left(\frac{g}{\frac{g * \cos\sigma}{\omega^2 A_y}}\right) = \cos\left(\frac{\omega^2 A_y}{\cos\sigma}\right) \tag{18}$$

Where: ***g*** —gravity acceleration [m/s^2]; ***σ*** — angle of the screen [°]; **ω** — angular frequency [rad/s]; ***A$_y$*** — amplitude of the screen in the y direction [m].

In Equation 19 we applied the relation given in 18 in the van Genuchten equation, where ***k*** factor is used to adjust this equation in all conditions of the system, its means that will work even when the bulk is on industrial dewatering and in a stockpile.

$$\theta = \theta_r + \frac{[\theta_{sat} - \theta_r]}{\left(1 + (\alpha_1 * k^{\frac{\omega^2 * A_y}{\cos\theta}} * |\psi|)^n\right)^m} \tag{19}$$

Below, in Equation 20 we are setting one equation equal to another in order to isolate a term u$_{eq}$ – equilibrium moisture.

$$\theta = \frac{u_{eq} \times (1 - \epsilon) \times \rho_s}{(1 - u_{eq}) \times \rho_l} = \theta_r + \frac{[\theta_{sat} - \theta_r]}{\left(1 + (\alpha_1 \times k^{\frac{\omega^2 * A_y}{\cos\theta}} * |\psi|)^n\right)^m} \tag{20}$$

This approach would allow us to take into account the characteristics of the bulk, such as the particle size distribution and the porosity, when predicting the dewatering rate/time. It would also allow us to consider the operating conditions of the vibrating screen, such as the frequency and amplitude. Below we try to unify both methodologies, considering that ***θ*** in van Genuchten equation is related to ***u***(***t***—>∞) = ***u$_{eq}$*** in King equation, adopting a matric potential corresponding to the applied ***G***, and additionally we introduce the term ***k*** to allow the equation to estimate the moisture when the system dewatering in a stockpile or similar element. With a few algebraic manipulations, the equilibrium moisture parameter can be made explicit, and then applied to King's equation.

$$u_{eq} = \frac{\left(\theta_r + \frac{[\theta_{sat} - \theta_r]}{\left(1 + (\alpha_1 * k^{\frac{\omega^2 * A_y}{\cos\theta}} \times |\psi|)^n\right)^m}\right)}{\frac{(1-\epsilon) * \rho_s}{\rho_l} + \left(\theta_r + \frac{[\theta_{sat} - \theta_r]}{\left(1 + (\alpha_1 * k^{\frac{\omega^2 * A_y}{\cos\theta}} \times |\psi|)^n\right)^m}\right)} \tag{21}$$

We adopt that,

$$V = \theta_r + \frac{[\theta_{sat} - \theta_r]}{\left(1 + (\alpha_1 * k^{\frac{\omega^2 * A_y}{\cos\theta}} \times |\psi|)^n\right)^m} \tag{22}$$

And,

$$H = \frac{(1-\epsilon) * \rho_s}{\rho_l} \tag{23}$$

By trivial algebraic arrangement, we have:

$$u_{eq} = \frac{1}{\left(\frac{H}{V}\right) + 1} \tag{24}$$

Finally, introducing Equation 24 in Equation 17, we have:

$$u(t) = \sqrt[(1-\beta)]{\alpha * (\beta - 1)t + \left(u_{in} - \frac{1}{\left(\frac{H}{V}\right) + 1}\right)^{1-\beta}} + \frac{1}{\left(\frac{H}{V}\right) + 1} \tag{25}$$

This new equation can forecast the ultimate moisture content while field force when the acceleration is ***G*** > 1.0, in case of industrial process using equipment like screens, and also when the field force ratio ***G*** = 1.0 (gravity draining), in case of stockpiles under dewatering. It tries to combine both materials conditions, a characteristic given by van Genuchten equation, and parameters that can model the effect of the industrial equipment on the granular material.

To ensure that new equation is precise enough is necessary to adapt the parameters to the operational condition with mineral, equipment and fluid features. This set up could be reached using approaches such as pilot tests, or even using a parametric methodology.

CONCLUSION

It may be possible to unify these two methodologies by combining their benefits and overcoming their limitations. As it shown, in theory, the new equation uses features from both methodologies to calculate the moisture content as a function of height, dripping condition, capillary forces and industrial conditions account field forces create by the equipment.

Considering the dewatering process is a complex operation in mineral treatment once it's a dependance of the water dynamics, time, operational conditions to better model this process is necessary a new approach. Both methods, King and van Genuchten, have independents characteristics that can improve the understand of dewatering of the bulk materials. As a final consideration, it should be noted that the empirical validation of the approach proposed here for real dynamic systems still depends on a robust experimental campaign, which is currently being developed using pilot-scale dewatering screens and will be the subject of future publications.

AKNOWLEGMENT

The authors are thankful to Brazilian Council for Technological and Scientific Development (CNPq), Foundation for Research Support of the State of Minas Gerais (FAPEMIG), Brazilian Federal Agency for Support and Evaluation of Graduate Education (CAPES), and Vale S.A. for their financial support. The authors also take the opportunity to declare that there is no conflict of interest regarding the publication of this article.

REFERENCES

ANDRADE, R. S. & STONE, L. F. Estimativa da umidade na capacidade de campo em solos sob Cerrado. **Revista Brasileira de Engenharia Agrícola e Ambiental**, v.15, n.2, p.111–116, 2011.

BENIS, S. G. **Performance study of a bench scale shale shaker** (Ph.D. thesis). Akron: The University of Akron. 2017. 218 p.

DUAN, C; YUAN, J.; PAN, M.; HUANG, T.; JIANG, H.; ZHAO, Y.; QIAO, J.; WANG, W.; YU, SH.; LU, J. Variable elliptical vibrating screen: Particles kinematics and industrial application. **International Journal of Mining Science and Technology**, 31 (2021) 1013–1022.

ETTMAYR, A., STAHL, W., KELLER, K., & SAUER, G. Dewatering of fine granular materials by vibrating screens with superposed capillary suction. Proceedings of the XXI International Mineral Processing Congress. Rome, Italy: Department of Chemical, Raw Material and Metallurgical Engineering. **Anais** […], Rome: University of Rome "La Sapienza". 2000.

FERREIRA, R. F. **Modelos para previsão do limite de umidade para transporte marítimo de finos de minério de ferro - TML**. 2019. Tese (doutorado em Engenharia Mineral). Universidade Federal de Ouro Preto, Ouro Preto. 2019.

FRADE, A. M.; LUZ, J. A. M. da. Optimization of screen dewatering through dynamic control of frequency. **Research, Society and Development**, [S. l.], v. 11, n. 7, e22711729823, 2022. DOI: http://dx.doi.org/10.33448/rsd-v11i7.29823.6.

GUERREIRO, F. S.; GEDRAITE, R.; SÁ, C. H. M.; ATAÍDE, C. H. Estudo de variáveis que afetam a umidade residual e a eficiência de separação em peneira vibratória. *In*: VI ENAHPE, 2015, Rio Quente. **Anais** [...]. 2015. DOI: 10.1590/s1517-707620220003000001

HAN, XIANG-WEI; SHAO, MING-AN; HORTON, R. Estimando parâmetros do modelo de van Genuchten de solos inalterados usando um método integral. **Soil Science Society of America Journal**, v. 73, n. 6, p. 2408-2417, 2009.

HE, D.; LIU, C.; LI, S. The Nonlinear Dynamic Behavior of a Particle on a Vibrating Screen Based on the Elastoplastic Contact Model. **Separations**. 2022, 9, 216. https://doi.org/10.3390/separations9080216

KELLER, K., & STAHL, W. Vibration screens for dewatering: theory and practice. **Minerals & Metallurgical Processing**, 27(1), 27-34. 1997.

KING, R. P. **Modeling and Simulation of Mineral Processing Systems**. 1st ed. Oxford: Butterworth-Heinemann. 2001.

LI, Z.; LIU, Y.; WANG, J.; LI, Y.; ZHANG, S.; WANG, X. Study on screening performance and parameter optimization of vibrating-dewatering screen. **Advances in Mechanical Engineering**, [S. l.], v. 13, n. 9, p. 1-15, 2021. DOI: 10.1177/16878140211046580. Disponível em: journals.sagepub.com/home/ade.

LOZANO, J. C., BLANCO RODRÍGUEZ, P., VERA TOMÉ, F., MALDONADO, R., MEDEIROS, A. S., & PRIETO, C. A system for obtaining soil solution extracts and soil water retention curves using a bench centrifuge with fixed angle rotors. **Geoderma**, 361, 114063. 2020.

LUZ, J. A. M. da. **Projeto "Drenagem acelerada de pilhas — minério de ferro" [internal report for Companhia Vale do Rio Doce — CVRD]**. Serra dos Carajás: Tecnomin Projeto e Consultoria Ltda. 1992.

LUZ, J. A. M. da. **Projeto “Drenagem acelerada de pilhas — carvão mineral”** [internal report for Copelmi Mineração Ltda.]. Butiá: Tecnomin Projeto e Consultoria Ltda. 1993.

MELLO, C. R. DE; OLIVEIRA, G. C. de, FERREIRA, D. F., LIMA, J. M. de, & LOPES, D. Modelos para determinação dos parâmetros da equação de van Genuchten para um Cambissolo. **Revista Brasileira de Engenharia Agrícola e Ambiental**, v. 9, n. 1, p. 23–29, jan. 2005. https://doi.org/10.1590/S1415-43662005000100004.

MOEZZIBADIA, M.; CHARPENTIER, I.; WANKOA, A.; MOSÉ, R. Temporal estimation of hydrodynamic parameter variability in stormwater constructed wetlands – The hysteresis effect during multi-rainfall events. **Ecological Engineering** 127 (2019) 1–10.

MUALEM, Y. A new model for predicting the hydraulic conductivity of unsaturated porous media. **Water Resources Research**, 12(3), 1976. Pp.: 513-522.

PATRA, A. S.; MAKHIJA, D.; MUKHERJEE, A. K.; TIWARI, R.; SAHOO, C. R.; MOHANTY, B. D. Improved dewatering of iron ore fines by the use of surfactants. **Powder Technology**, 287 (2016): 43–50.

QUITES, N. A. S. **Influência de diferentes tipos de minério de ferro na umidade do *sinter feed*** (Dissertação de mestrado). Universidade Federal de Ouro Preto, Ouro Preto, 2018.

RAJA, V., CHASE, G. G., JONES, B. N., GEEHAN, T. Continuum model of a shale shaker. *In*: Proceedings of the 2010 AADE Fluids Conference and Exhibition (pp. 1-10). **Anais** […] Houston, TX: American Association of Drilling Engineers. 2010.

RUDIYANTO, M., SAKAI, M., VAN GENUCHTEN, M. T., ALAZBA, A. A., SETIAWAN, B. I., & MINASNY, B. A complete soil hydraulic model accounting for capillary and adsorptive water retention, capillary and film conductivity, and hysteresis. **Water Resources Research**, 2015, 51, doi: 10.1002/2015WR017703.

SCARPARE, FÁBIO VALE E MORAES, SÉRGIO OLIVEIRA; DE JONG VAN LIER, QUIRIJN. **Disciplina LCE200: Física do ambiente agrícola — Umidade do ar**. Piracicaba: Escola Superior de Agricultura Luiz de Queiroz, Universidade de São Paulo.

HENG-SHEN, Y., JIAN, D., & MING-HONG, Z. (2002). Research of regularity of throwing movement for solid particle on the Shaker’s screen. **Applied Mathematics and Mechanics**, 23(6), 655–659 (2002). doi:10.1007/bf02437649.

STOICOVICI, D., UNGUREANU, M., UNGUREANU, N., & BANICA, M. Computer Model for Sieves' Vibrations Analysis, Using an Algorithm Based on the False-Position Method. **American Journal of Applied Sciences** (2009), 6, 48-56. https://doi.org/10.3844/AJASSP.2009.48.56.

TSCHAPEK, M.; FALASCA, S.; WASOWSKI, C. The Undrainable Water in Quartz Sand and Glass Beads. **Powder Technology**, 42 (1985), 175-180.

WANG, Z. M., ZHAO, Y. B., ZHANG, C. X., DENG, M., & ZHANG, D. H. Simulation of flow through vibrating screens with moving body method. *In*: 2010 International Conference on Computer Application and System Modeling (ICCASM 2010). **Anais** […] Shanxi: IEEE, 2010.

YU, C.; WANG, X.; PANG, K.; ZHAO, G.; SUN, W. Dynamic Characteristics of a Vibrating Flip-Flow Screen and Analysis for Screening 3 mm Iron Ore. **Shock and Vibration**, [S. l.], v. 2020, Article ID 1031659, 12 p., 2020. https://doi.org/10.1155/2020/1031659.

ZHANG, Y., ZHAO, D., ZHENG, Q., HUANG, Y., JIANG, F., WANG, M.-K., LIN, J., & HUANG, Y. Evaluating the effects of temperature on hydraulic and mechanical properties in the collapsing gully areas of South China. **Catena**, v. 218, p. 106549, 2022.

ZHU, W.; WU, W.; HAO, X.; WANG, H. A Novel Solution to Sieving Model for a Drilling Fluid Shale Shaker. **Telkomnika Indonesian Journal of Electrical Engineering**. v. 10, n. 6, October 2012, pp: 13578 – 1363.